of the pressure $-\sigma$, when the volume of the cube changes. The term $[(1-2\nu)/E]\,\mathrm{d}\,(s_1+s_2+s_3)\,\sigma$, the meaning of which cannot intuitively be caught, can be done away with by putting down : $s_1+s_2+s_3=0$, that is to say by putting down $\sigma=\frac{1}{3}(\sigma_1+\sigma_2+\sigma_3)=\sigma_m$. By making use of following identity : $0=(s_1+s_2+s_3)^2=s_1^2+s_2^2+s_3^2+2(s_1s_2+s_2s_3+s_3s_1)$ one obtains dW's final expression

$$\mathrm{d}W=\frac{1+\nu}{2E}\,\mathrm{d}\,(s_1^2+s_2^2+s_3^2)+\frac{3\,(1-2\,\nu)}{2E}\,\mathrm{d}\sigma_m^2$$

= distortion energy + volume change energy.

This relation shows, that, in order to obtain the pure distortion energy, the average stress σ_m has to be substracted from stresses σ_1, σ_2 and σ_3 and that the sole residual stresses s_1, s_2 and s_3 are to be taken into account. The finite form of the distortion energy is obtained by integrating dW from an initial unstressed state up to a final one, characterized by s_1, s_2, s_3. Continuing to call the sole distortion energy by W, one obtains following relation

$$W=\frac{1+\nu}{2E}\,(s_1^2+s_2^2+s_3^2).$$

In the case of a thick-walled cylinder, one extracts from eqs. (9), (10) and (12) following relations

$$-s_r=s_t=\frac{p_1-p_2}{k^2-1}\,\frac{k^2}{l^2}\quad\text{and}\quad s_z=0$$

so that the specific wall distortion energy is expressed as follows

$$W_l=\frac{1+\nu}{E}\left(\frac{p_1-p_2}{k^2-1}\,\frac{k^2}{l^2}\right)^2.$$

Subindex l has been added to W, in order to draw the attention to the fact, that the energy varies with l. Considering the most frequent case of wall submitted to an internal pressure only ($p_2=0$), the greatest energy is to be found near the bore of the cylinder ($l=1$)

$$W_{l=1}=\frac{1+\nu}{E}\left(\frac{p_1\,k^2}{k^2-1}\right)^2.$$

Considering now a tensile test-piece, which has reached the upper yield stress σ_y ($\sigma_1=\sigma_y$, $\sigma_2=\sigma_3=0$) one finds $s_1=\frac{2}{3}\sigma_y$ and $s_2=s_3$ and a maximum uniformly distributed distortion energy in the test-piece

$$W_y=\{(1+\nu)/E\}\,\tfrac{1}{3}\,\sigma_y^2.$$

As Maxwell did, let us now assume, that all the bodies of a same material cease to have elastic reactions at places where and at the moment when their distortion energy reaches a critical level. The critical level of the cylinder energy $W_{l=1}$ must be then equal to the maximum energy W_y of the tensile test-piece, so that the critical pressure p_{1y} is a function of the upper yield stress σ_y and is expressed by following relation

$$p_{1y} = (1 - 1/k^2)\, \sigma_y / \sqrt{3}. \tag{14}$$

Eq. (14) is applicable, as shown in following paragraph.

5. The Plastic Flow of a Thick-Walled Cylinder

By gradually increasing the pressure, applied inside the cylinder, the material layers at the bore show first an elastic deformation, then a plastic one, which begins, when the pressure has reached a critical level p_{1y} called the yield pressure. The plastic deformation propagates through the wall, under increasing pressure, until the whole material has become plastic, which happens, when the pressure has reached a level p_{1c} called the "collapse" pressure. The constant pressure p_{1c} produces greater deformations in ductile materials, characterized by a small k ratio, until the material layers at the bore begin to harden. Then the pressure must be increased, in order to increase the deformation, because a more and more important part of the material hardens. At the moment when the deformed wall thins down and simultaneously hardens, so that the two phenomenons are compensated, the pressure applied, reaches its peak level. This maximum pressure p_{1u} is called the ultimate pressure. The wall swells rapidly at a determined place and bursts at a pressure, which is lower than p_{1u} and is of little interest. The wall remains thus cylindrical, until pressure p_{1u} is reached.

There is a pressure range, which is particularly interesting to study, and it is this one, comprised between pressures p_{1y} and p_{1c}, in other words, the range, in which the deformations produced cannot freely develop and as far as their smallness is concerned, are still comparable with elastic deformations. When the pressure is going down, the wall portion, submitted to an elastic deformation shows a tendency to reassume its initial shape but it is hindered from doing so by the underlying wall portion, which has been overstrained. Residual stresses consequently appear, which are to a certain extent similar to these appearing in an elastic wall, reinforced by shrinkage. This is the reason why a wall made of a material, which has been partially submitted to plastic deformations is called "self-hooped" or